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ABSTRACT

This document begins by defining and discussing educational planning. A brief overview of mathematical programing with an explanation of the general linear programing model is then provided. Some recent applications of mathematical programing techniques to educational planning problems are reviewed, and their implications for educational research methodology are noted. Applications of linear programing are given at national, State, and local levels, with emphasis on problems at the local school district level. The document concludes with some directions and implications for future research, and a discussion of mathematical programing applications in light of recent developments in educational programing-planning-budgeting systems. (Author/RA)

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MATHEMATICAL PROGRAMMING MODELS IN EDUCATIONAL PLANNING

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No man is an island--and no profession can remain isolated from the ferment in its environment. To remain a viable force in education, administrators must be sensitive to new developments and be capable of adapting the relevent technology of this era to their tasks. (AASA Commission on Administrative Technology-1963.)

During the past few years there has been a rapid development of the field called management science which should not be confused with the older tradition of "scientific management" in the sense of time and motion studies (Fox, 1967). Management science models which express the organizational environment and its dynamics in mathematical relationships have been applied in the fields of business management, military operations research, public welfare and regional planning. Although the value of such models has been demonstrated in the areas listed above, educational administrators and planning specialists appear to be making little use of these scientific approaches.

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Reasons for the gap in education can be found in a number of recent writings (Sisson, 1969; Lachene, 1969; Bowles, 1970). These and other explanations are introduced at appropriate points within the review so that the proper emphasis can be placed on methodological considerations for future applications. Commenting on the value of these management science models, Fox (1967) notes that not all aspects of academic life are or should be subject to quantification; however, he claims that many decisions are made by administrators that affect in quantitative ways the "input" and "output" of the educational process. The research reported in Bereday and Laufwerys (1967) and Correa (1969) provides similar conclusions about the nature and scope of quantitative methods in educational planning.

In light of the prevailing emphasis on systems analysis in education, there is an emerging literature that advocates the use of management science and operations research models as a means to increase the efficiency of educational planning and decision-making (Knezevich, 1969). Too often, however, the intent of these articles is to focus on the advantages of applying models rather than to provide empirical research that illustrates the unique contributions of such models to generate solutions for real and immediate educational problems.

Hence, there exists a need for educational planners to conduct studies that determine the utility and limitations of management science models to aid in improving educational planning and decision-making. In this review I provide a number of recent studies that illustrate how a particular management science tool, namely, mathematical programming,

might be used by educational planners to provide a new approach for solving various educational planning problems.

The review is structured as follows. First, a discussion of educational planning, including a definition that provides a framework for illustrating the limits of the review, is presented. This is followed by a brief overview of mathematical programming with an explanation of the general linear programming model. The explanation of the model should acquaint the reader with the basic terminology used in mathematical programming. Next, some recent applications of mathematical programming techniques to educational planning problems are reviewed and the implications of this work for educational research methodology are noted. Applications are given for selected problems at national, state, and local levels of planning. Special attention is given to applications at the local school district level, since most applications in this domain are of recent origin and have been developed in disciplines other than education, i.e., industrial engineering, economics, business and operations research. In the final section I provide some directions and implications for future research. This includes a discussion of future applications of mathematical programming in light of recent developments in educational-program-planning-budgeting.

EDUCATIONAL PLANNING

Chirikos and Wheeler (1968) have noted that despite the rapidly expanding number of writings on the subject, there is at present little agreement on the dimensions or scope of educational planning.

Characteristic of most recent descriptions of educational planning, however, is that it is a process of preparing information in the form of a set of alternatives (with estimates of their specific consequences) to aid decision-making for management policy formation and administrative action. This represents a significant departure from the "traditional" notion of planning that focuses on a method for carrying out a pre-determined design or master blueprint (Wolvek, 1968).

Based on a study to determine the basic planning principles to serve as criteria for educational planning efforts and projects, the Iowa State Department of Education provides the following operational definition:

> Comprehensive educational planning is a process to produce valid information in the form of alternative courses of action, together with predicated consequences of such alternatives, to aid decision making by those engaged in educational policy formulation and administration. The process should be capable of providing information relative to any educational problem input and should incorporate self renewal and updating as essential features. (Wolvek, 1968; page 9)

Furse and Wright (1968) provide an excellent glossary of educational planning terms as well as a rigorous treatment of each discrete component in the planning process ranging from the initial conceptual design to the final evaluation phase.

The distinction between human resource planning and educational planning is well treated in Davis (1966, Chapter 1). Human resource planning attempts to clarify relationships of the educational system to economic and social development (Parnes, 1962). Educational planning, on the other hand, focuses primarily on the internal operation and

management of schools. Failure to make this distinction accounts for a portion of disagreement among researchers and planners as to the objectives and goals of educational planning. This is especially true when educational researchers attempt to utilize the results of research studies published in the economic literature and vice versa.

Since the primary purpose of educational planning is to design educational policy, studies in this area include such diverse topics as curriculum selection, design of physical facilities, resource allocation strategies, cost accounting, salary schedule analysis and student population projections (Chirikos and Wheeler, 1967; Correa, 1969). This review focuses on the application of mathematical programming techniques to these types of problems. Cronbach and Suppes (1969, pp. 20-21) refer to this category of research activity as "decision-oriented research." The review does not treat the application of these techniques to the psychological aspects of learning. The reader interested in this area might first turn to the research described in Restle (1964).

MATHEMATICAL PROGRAMMING

Mathematical programming is a quantitative approach where by some criterion of effectiveness (known as the objective function) is maximized or minimized subject to a set of constraints. Mathematical programming consists of many various optimization techniques. Some of the most widely used methods are (1) distribution methods such as stepping-stone or modified-distribution approaches, (2) Simplex method or (3) approximation methods (Metzger, 1963).

Although the terms mathematical programming and linear programming seem to be used interchangeably in the literature, mathematical programming is a generic term. It includes both linear and nonlinear programming as well as integer, heuristic and dynamic. Since most applications of mathematical programming to educational planning problems have used linear programming techniques, the general linear programming model is presented below. This should serve to acquaint the reader with some concepts that are discussed in relation to the methodological issues.

It is my position that many educational planning problems might better be approached using nonlinear programming techniques such as dynamic or chance constrained programming rather than linear programming methods. Hence, I briefly describe a number of nonlinear programming approaches in this section and provide the reader with a standard text which can be used for further investigation in each area.

Linear Programming

Linear programming is basically a mathematical technique that can be characterized by:

1. A linear equation that expresses the relationship between an appropriate quantity such as cost and the basic parameters of the system.

2. A set of linear equations or inequalities that express the constraints of the system or the variable interactions in the system (Bruno, 1969a; p. 485).

The above can be represented mathematically by the following system of equations and relationships. Let:

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = Z \qquad (1)$$

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m
\end{aligned} \qquad (2)
$$

and

$$x_i \geq 0 \quad i = 1, 2, \ldots, n. \qquad (3)$$

The objective function is represented in (1). The activity constraints are represented in (2). The non-negativity constraints in (3) require that the solution values of each variable be either zero or positive but not negative. This is necessary because most educational planning variables, e.g., allocation of dollars to programs, number of students enrolled, etc., usually have no negative counterparts.

In vector notation, the linear programming model above is represented by:

$$
\begin{aligned}
CX &= Z && \text{Objective Function} \\
AX &\leq B && \text{Constraint Set} \\
X &\geq 0 && \text{Non-negativity Restrictions}
\end{aligned} \qquad (4)
$$

where:

C = a row vector of dimensions 1 by n containing the coefficients of the objective function.

A = an mxn matrix of coefficients for the variables in the constraints set.

X = a column vector of dimensions n by 1 representing the variables in the system.

B = a column vector of dimensions m by 1 containing the right side of the constraint set

Z = a scalar representing the value of the objective function.

All linear functions by themselves have a maximal value of positive infinity and a minimal value of negative infinity if no restrictions are placed on the value of their variables. Consequently, the problem of optimizing a linear function (objective function in our case) is not a mathematically meaningful problem unless the variables are constrained as to the ranges of values they may assume. Constraints not only provide regions of acceptable values of the variables but also provide a mechanism for relating the mathematical expressions to real world conditions. For example, constraints can be used to reflect realistic time and material considerations imposed by planning schedules or financial restrictions imposed by a budget.

Four points should be kept in mind in connection with the constraint set described above. First, the coefficients of A and B in (4) are all simply constants. Second, all constraints in the general model illustrated above are linear in the same sense as the objective function. Third, the constraints are characteristically inequalities although equality-constraints are not categorically excluded. Finally, the number of constraints, i.e., m, is not restricted in any way except as it affects the practical problem of computation.

As a system the constraint set provides a feasible solution area in n-dimensional space. Since this feasible solution area or range of acceptable values of the variables may or may not include values up to infinity, linear programming becomes a problem of selecting a set of values (of the variables) within this region, which will yield the optimal (maximal or minimal) value of the objective function.

Three definitions can be offered in summary. Optimization is the process of finding a best solution among several feasible alternatives. The term "best solution" is used because there may be more than one optimal solution. Variables are those factors that can be manipulated to achieve the desired objective. In programming parlance these variables are commonly referred to as independent or decision variables. They are the x values which form the column vector X in (4). Parameters are those factors that effect the objective function but are not controllable, i.e., they can not be manipulated as are decision variables. These include all the constants in A, B, and C described in (4) above.

Parameteric programming techniques are available that allow for the manipulation of one or more of these parameters. Parameteric linear programming has been used in a number of educational planning models (Bowles, 1969; Bruno, 1969b). These techniques are usually employed as part of the post optimal sensitivity analysis that is used to get the maximum value from optimal solutions generated by linear programming models (Ferrero di Roccaferrera, 1970).

Gaus (1970) has written an excellent elementary textbook on linear programming that can easily be read by the novice. He provides a good coverage of many other standard linear programming techniques such as the transportation model, the personnel-assignment model, the maximal flow network model, and so on. Both Banghart (1969) and Davis (1969) provide a brief overview of the relationship of the various linear programming models to educational planning appropriate for the same

beginning target population.

Nonlinear Programming

In general, programming problems in which either any term of the objective function or any one of the constraints is represented by a nonlinear function can properly be called a nonlinear programming problem. Suppose the objective is to

$$\text{Maximize } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} \qquad (5)$$

where $c_{ij} = f_{ij}(x_{ij})$ and f_{ij} is a linear function of x_{ij}. Then Z is a quadratic function. When the objective function is nonlinear (quadratic or more complex) and the constraints are linear, the model can often be formulated as a convex programming problem (McNamara, 1970).

The class of nonlinear programming problems which has been utilized most extensively is that in which the constraints are linear and the objective function is nonlinear. Special cases of particular interest, dealing with separable objective functions and several decision variables can be found in Hadley (1964). It should however be mentioned that an arbitrary function of n variables is not separable. Closely related to the notion of separability is single-stage versus multi-stage modeling. Computational advantages in this area as well as the relationship of the approach to decision-making structures are treated in Nemhauser (1967).

Another class of nonlinear programming problems that is of special interest to educational planners is obtained from the general linear

programming problem by imposing the additional requirement that the variables can take only integral values. Such problems are frequently referred to as integer linear programming problems. Integer programming methods are often used in problem areas such as scheduling where integers must be used to represent student assignment to courses.

Dynamic Programming

Often, following the construction of a mathematical model of an educational planning problem, but prior to performing the optimization, it is desirable to make some changes of variables and transformations. Basically, dynamic programming is such a transformation (Nemhauser, 1967). It takes a sequential or multi-stage decision process containing many interdependent variables and converts it into a series of single-stage problems, each containing only a few variables. Such transformations are invariant in that the number of feasible solutions and the value of the objective function associated with each feasible solution is preserved (Bellman, 1957).

Many educational planning models encompass a number of time periods so as to allow consideration of the intertemporal relationships within the educational system. Bowles (1967) has successfully applied dynamic programming methods to solve his planning model on a year-by-year sequential basis. He notes

> Operation of a model in this manner is probably a good reflection of the actual policy-making process which proceeds from year to year rather than on a once-for-all basis for an entire n-year period. In addition it allows the efficient use of new data. A further advantage is that it avoids the necessity of acting on the values of the instrument variables in the later years in the planning period, which are presumably sensitive to somewhat arbitrary conditions (p. 194).

Dynamic programming is a technique that holds great promise in the area of "decision-oriented" research in education. Such techniques allow for more realistic modeling of the educational decision-making process. This is particularly true in cases where long and short term educational plans must be reconciled (McNamara, 1970; p. 47). Dynamic programming is sometimes referred to as recursive optimization which is a more representative but less glamorous name for this approach.

Stochastic Programming

In some educational planning models the coefficients in the objective function or the other parameters in the constraint set might best be represented by random variables. In this case the programming problem involves a stochastic model. There are a number of methods that have been applied to stochastic programming problems (Senupta, 1966). The active and passive approaches to stochastic programming could be applied. Chance constrained programming is a likely candidate, since this approach includes the possibility that a constraint is violated.

In chance constrained programming, a constraint from the general linear programming model given in (6) below

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \leq b_i$$

or in summation form

$$\sum_{j=1}^{n} a_{(i)j}x_j \leq b_i \qquad (6)$$

could be written as

$$P\left(\sum_{j=1}^{n} a_{(i)j}x_j \leq b_i \right) > \alpha_i \qquad (7)$$

where α_i is a probability not far from 1.0. In the general linear programming model, the α_i's have been set at 1.0.

The general linear programming model is a deterministic case in the chance constrained programming format. This follows since a deterministic model can be considered as a special case of a stochastic model in which each random variable assumes a particular value with probability 1.0 and all other values with probability zero.

In the remaining part of the review, the emphasis moves away from a discussion of different optimization strategies and toward a review of applications of these approaches to specific educational planning problems. For simplicity and continuity, these applications are grouped using a geographic orientation. Specifically, the three subdivisions are based on models which focus on problems at the national, state, and local levels of planning. Some national and state level applications have been reported in the economics (and to a lesser extent in the educational)literature, i.e., in the areas of developmental economics, human resource planning and comparative education. Applications at the local or micro level are seldom found. Hence, a greater emphasis is placed on a more extensive review of this literature. Macro level examples used here are to a large extent illustrative. They are included to provide the reader examples of how such models and techniques might be adapted to appropriate decision-making situations in a micro setting such as a school district, intermediate unit, or university.

Applications have been selected where possible from very recent research with a view toward presenting, in a synoptic fashion, research studies conducted within the disciplines of (1) engineering, (2) regional science, (3) operations research, (4) economics and (5) educational

administration. Special attention has been given to reports just now emerging from places such as Colleges of Engineering where there has been (for obvious reasons) a movement toward model building in education. Earlier applications of mathematical programming in educational planning have been reviewed by Fox and Sengupta (1968) and Chirikos and Wheeler (1968).

MACROANALYTIC APPLICATIONS: THE NATIONAL LEVEL

The large amount of resources devoted to education and the increasing emphasis to design educational policy in relation to an overall set of objectives for economic and social development (Sanders and Barth, 1968), has resulted in the creation of educational plans in virtually all the major nations of the world. The literature on the subject abounds with such concepts as "human capital" and "investment in man," attesting to the growing realization that education and training are every bit as important in contributing to productivity capacity as raw manpower and physical capital (Parnes, 1962).

The growth of actual planning operations has been paralled by the rapid increase (primarily in academic communities) in utilizing quantitative approaches such as mathematical programming to analyze basic economic characteristics of the education system (OECD, 1965, 1967, 1969). These characteristics include its internal productive relationships and the nature of the demand for its output in the economy. Since mathematical programming allows the planner to view an educational system as a set of input-output or production relationships, which can be controlled in a way that will optimize the use of scarce educational resources, it be-

comes a valuable technique to generate policy or "decision-oriented" information which provides new knowledge to the decision-maker about consequences of certain feasible alternative strategies.

Most common educational planning models derived at the national level are based on the "social demands approach" and/or the "manpower-requirements approach" (Golladay, 1968; Chapter Two). Briefly, the "social demands approach" to educational planning attempts to project individuals' demands for places in the educational system as opposed to society's demand for trained manpower. The "manpower-requirements approach" ascertains future needs for manpower from projections of the growth of the economy and imputs of labor of various skills; the resulting needs of the economy are compared with expected supplies of labor to determine production goals of education (Williams, 1969; Levasseur, 1969).

Most mathematical programming techniques have been applied in manpower planning models, however, a number of these models have attempted to treat social demands in some implicit fashion, i.e., in the formation of the parameters in the objective function or in the constraint set. Discussion of the implications of using (1) the supply-demand criterion, (2) optimization techniques in manpower planning and (3) data needs for such models in educational planning are all treated in McNamara and Franchak (1970). The economist or model builder who wishes to assist educational administrators in decision-making by providing predictive models which measure the effect technological change and its implications for occupational education planning should consult the recent

research reported in Kraft (1970) and Venn (1970).

Programming models with investment in education as a component of aggregate national investment have been reviewed in Fox and Sengupta (1968). Common to this type of model are (1) use of a dynamic programming approach with a planning horizon of eight to ten years, and (2) simultaneous optimization of investments in education and real capital for different sectors of the economy (Adelman, 1964).

Bowles (1969) has developed a dynamic linear programming model for the educational system in Northern Nigeria which uses available information to determine the values of instrument variables for an eight-year planning period. The instrument variables include enrollments and resources used at the various educational levels (primary, secondary, etc.) in each time period. The instrument variables have been defined to correspond to the actual policy instruments available to most governments. The generalizability of this model is evidenced by its subsequent use in Greece and Canada.

The objective function of the Bowles model is used to maximize the present value of the contribution of various educational levels to future national income. Constraint categories reflect school policies regarding items such as (1) types of teachers, (2) student enrollments and transfer among the various types of schools, (3) school construction and current facility usage, (4) teacher recruitment or importation of foreign instructors and (5) political or legal restrictions.

Golladay (1968) has constructed a similar macro level model for Morocco in which the educational system is represented in the form of a

intertemporal input-output model. The educational production functions (objective functions) are the conventional linear Leontief type and the economy is modeled as a modified, dynamic Leontief system. Other such models and the methodological implications of this type of activity can be found in Davis (1968). The use of sensitivity analysis to derive a series of important policy prescriptions that are not easily revised by any plausible degree of error in the data or in the model assumption is well treated in Bowles (1969, pp. 130-133).

One obvious instrinic property of these macroeconomic educational planning models is that they treat a national educational system at its highest level of aggregation. Hence, within a single model, the objectives for primary, craft, technical and secondary schools in addition to universities are simultaneously optimized. This property, coupled with the reality that education is basically a state function (a decentralized system for control and management), explains in part why such models are seldom developed in this country.

Although the objective functions of these macro planning models do not directly coincide with the objectives of educational decision-makers at the local level, these approaches to planning do provide a method for modeling instrinsic properties of local schools, i.e., constraints dealing with teacher assignments, student flow matrices, and instructional resource allocations to specific programs within a school. Or put in methodological terms, although mathematical programming is used to optimize preference functions of decision-makers, a large variety of other mathematical techniques are used in conjuction with

programming algorthims to form realistic and explicit relationships for the systems constraints. For example, dynamic programming is sometimes used with markov chains and integer programming is often used in conjuction with network theory.

MACROANALYTIC APPLICATIONS: THE STATE LEVEL

The structure of the United States educational system can be briefly summarized as a federal-state-local partnership. This section deals with educational planning models developed for State Departments of Education (SDE's). In general,research in this area suggests that SDE's should have less preoccupations with regulations and more concern with leadership. Specifically, these leadership functions should include planning services that are relevant to the needs of local school systems and should be based on systems management rather than merely reflecting experiences in school administration (McNamara and Smink, 1970).

Vocational Education

In an attempt to expand their current leadership activities in educational planning, many states have recently developed planning data basic to their role in helping local school districts and other educational institutions or training agencies within their state to establish better planning capacities or to cope with long range planning problems. In light of the requirements of the new vocational education act, P.L. 90-576 The Vocational Education Amendments of 1968, State Boards of Education are required to submit state plans for vocational technical education programs. States are required to project, by various occupational categories, labor market supply and demand for the current year and for a target year five years following.

One immediate consequence of this legislation is that planning models developed for investigating long-term funding policies of SDE's, can now use standardized data inputs. Moreover, mathematical programming models developed for national policy information by research agencies such as the Organization for Economic Cooperation and Development (OECD, 1965, 1967, and 1969) could be adapted by SDE's to determine the relative merits of alternative vocational education policy decisions in light of economic and social developments.

Section 123 of the Vocational Education Amendments of 1968 dealing with new state plans makes it clear that each state will be expected to develop a state-local planning procedure that will assure the best use of funds in light of important training needs of all people as well as requirements of employers. McNamara (1970) has developed a linear programming model designed to be used by State Boards of Education in conjunction with their state-local program plan to evaluate decisions about the efficient allocation of vocational education funds to local school districts.

The McNamara model is based on the manpower-requirements approach jointly developed by the Office of Education and the Manpower Administration of the U. S. Department of Labor (U. S. Department of Health, Education and Welfare, 1969). The objective function of this model is used to determine the maximum number of additional graduates who can be trained in occupations for which there currently exists critical shortages in the labor force. The constraints are specified by occupational program by county in a specific labor market area. These include

(1) labor market area constraints based on critical occupational shortages, (2) budget constraints that are adjusted to reflect attrition rates between enrollments and graduates and (3) school capacity constraints by specific programs and by total expected enrollment growth in each county.

For methodological and illustrative reasons, three specific properties of this model deserve further mention. First, since labor market areas are federally defined subdivisions, and since schools across the country exhibit similar organizational characteristics, this type of model provides a generalizable procedure that can be used by any state. Second, research has shown that data currently assembled by different state government agencies is used almost exclusively for accounting and reporting purposes. This model illustrates how these statistics collected by state government agencies, i.e., Departments of Commerce, Labor and Education, etc., can be transformed into meaningful planning information.

Finally, although the planning model is linear and not intrinsically dynamic, procedures are given to treat the time factor and the long-term consequences of particular funding decisions. This is accomplished by supplying the decision maker with additional information based on the optimal solution and by the use of student attrition rates in the budget constraints.

Balinsky (1970) has developed a set of dynamic feed-forward models to provide analytical solutions for manpower planning models. Optimal closed form solutions minimizing given objective functions of manpower models are provided. The models considered treat people as the only flow

variable. The flow originates with some particular population of eligible persons and then traces them through the educational and economic sectors of the model. A number of different mathematical programming techniques are used throughout the study. Other recent studies in this area include the recursive programming model of Maki (1970) and the linear programming model developed by Patz (1970).

State Finance Formula

In order to determine optimal resource allocation strategies Bruno (1969a) has developed a linear programming model to be used with foundation type state support programs. This model can be solved under various objective functions and constrained by different "design criteria" related to variance in per pupil expenditures for those districts in the program.

In a subsequent report of this research, Bruno (1969c) used parametric programming techniques to generate additional policy-type information about minimum state aid available and total state funds allocated. The important contribution of this approach is that it realistically places the problem of educational finance in a system constrained by political,social and economic considerations.

Most current research in state school finance has been conducted with apparent disregard for any constraining conditions that might be placed upon state resources available for distribution. Some research shows no evidence of moving toward new approaches to state level allocations but rather is concerned primarily with modifying present state support formulas.

Resource allocation models, based on mathematical programming, provide a potential new direction for this type of research. For example, school district boundaries often coincide with other geographic divisions of states such as standard metropolitan statistical areas (U. S. Department of Commerce/Bureau of Census) and labor market areas (U. S. Department of Labor). In such cases constraints for financial optimization models, based on current characteristics given in these demographic reports, might be used to form constraints in a state support system that better reflect regional social and economic differences among school populations.

State-Wide Facility Planning

Hopkins (1970a) has used linear programming techniques to determine the optimum location of vocational-technical training centers to form a state-wide system of training centers for the State of Oklahoma. The objective function of this model is used to minimize the average miles traveled by each student. Constraints are used to restrict distance traveled and the number of training centers to be constructed in light of the optimum use of existing facilities.

Hopkins (1970b), a report with the same title, is an excellent illustration of how a non-technical presentation of a linear programming study can be constructed for decison-makers. Mathematical relationships are illustrated either by maps or appropriate data tables rather than by formulas. Similar geographic optimization models have been used for studying school district rezoning in light of current desegration problems. These are reviewed in a subsequent section dealing with micro-analytical applications.

Eastman and Kortanek (1970) have shown that the demand for educational facilities is dependent on the mix of housing designs and their sequence of construction. A markov model, cast in a linear programming format, is developed that explores the relationship between alternative growth curves of student population and the housing mix and construction sequence. Objective functions and constraints are formulated that allow for a wide variety of school planning objectives to be related to the housing mix and schedule as defined by market, social, economic or administrative constraints.

With the current trend toward (1) fewer but larger, more comprehensive school districts (Cohn, 1968), (2) state supported intermediate units with centralized instructional resources, and (3) new area vocational-technical centers which usually are designed to service a large number of school districts within a county or regional area; there exists a large domain where any number of geographic optimization models can be used to increase the efficiency and effectiveness of state level educational planning. The application of these models to a number of diversified problems of public administration can be found in the various issues (1968 to present) of *Socio-Economic Planning Sciences*.

MICROANALYTICAL APPLICATIONS: THE UNIVERSITY LEVEL

Higher education is currently a ten billion dollar a year enterprise that includes more than six million students, faculty, and staff. Managers in higher education have recently turned to program budgeting and analytical models to sharpen their control and understanding of

their institutions and of the effects of their decisions on the course of such a massive and complex enterprise (Weathersby and Weinstein, 1970). A limited number of these are discussed below.

Universities have made far more progress in applying mathematical programming and other management science models to academic administration than have local school districts (Hufner, 1968). The recent illustrations presented here should point out some specific problem areas that school district planners might consider for using programming techniques. Guidelines and decision-making parameters that should govern the application of optimization models in American universities are well summarized in Barton (1970).

Department Level Management

McCamley (1968) has used activity analysis, a form of linear programming to determine optimum output and activity levels of, first, a department within a college, and then for a hypothetical college containing three departments. His model contains forty six different commodities (departmental services) in the activity vectors. Departmental outputs include degrees conferred, teaching to support students in other departments, three types of research activities and publications.

Fox (1957) has used a linear programming model for allocating a given faculty among alternative teaching and research assignments. Recursive and dynamic programming are used to extend this effort to compute optimal decisions over a sequence of years. A two-level decision model involving interaction between a dean and department chairmen in planning resource allocation is also provided in this study. Briefly,

this problem centers around the following organizational problem. The objective functions for the various departments at one organizational level are all viewed by the dean as constraints, i.e., each competing for part of his resources. However, the dean also has an objective function which involves maximization across all departments in his jurisdiction. This problem is handled by using a two-stage optimization model. Part of this research is also described in Plessner, Fox, and Sanyal (1968).

Turksen and Holtzman (1970a) have recently developed mathematical programming models related to departmental levels in the organizational structure of a university. Their programming formulations are in linear and quadratic integer forms. These formulations are extended into dynamic programming for finite short range planning purposes. Based on this and other previous work, they have also constructed an iterative dynamic programming model to facilitate short term operational planning and forecasting (Turksen and Holtzman, 1970b).

Andrew (1970) has designed a linear programming model for matching available faculty in a department to courses that must be taught in a particular semester. Faculty are assigned to scheduled courses based on measures of their effectiveness and teaching preference for each course offered. Flexibility for the decision-maker is built into the objective function by allowing a to be the weight of teacher preferences and $(1-a)$ the weight of the effectiveness criterion. Thus, if an administrator wished to place a weight of 0.35 on teacher preference, 0.65 would be the weight on teacher effectiveness in the system. This type of model could be easily adapted by a secondary school department.

Systems Level Planning

Least cost decision rules for transferring library materials from primary to secondary storage have been developed by Mann (1970) using a dynamic programming model of a university library system. Descriptions of extensive work in modeling libraries, including considerations of acquisition, circulation, demand and associated cost can be found in Morse (1968). Crandell (1969) has used linear programming to build a constrained choice model for student housing. The purpose of this model is to aid in a university's decisions regarding both construction of and subsidies for student housing.

Application of different mathematical programming techniques for university management have been developed in connection with the Generalized University Model Project at the University of Texas (Sen, 1969; Schoeman, 1969). Mathematical programming has also be used in a number of university management studies developed in connection with the Research Program in University Administration coordinated in the Office of the Vice President--Planning and Analysis, University of California (Bartholomew, 1969; Sanderson, 1969; Weathersby, 1970a).

Another mathematical programming technique, optimal control theory (Tracz, 1969), has recently been used by Weathersby (1970b) to investigate the allocation of public resources in higher education. The relationship of optimization models and simulation techniques for university planning is discussed in Weathersby and Weinstein (1970). Basically, simulation models attempt to associate cause with effect and these models

do not require the explicit objective functions needed for optimization models.

Programming models developed for university management might be adapted for public elementary and secondary school management in larger school districts characterized by (1) a large set of diversified instructional facilities and resources, (2) a complex organizational hierarchy, and (3) a multi-level program-planning-budgeting process. In each case, these models could be used to explicitly formulate a control process, or (without specific mathematical relationship) to evaluate current management procedures.

MICROANALYTIC APPLICATIONS: SCHOOL DISTRICT LEVEL

The burdens placed on public schools are growing in number and complexity. More and more people are turning to educational institutions to help solve the social problems of our times. These include such items as poverty, racial tension, occupational obsolesence in the work force and a scarcity of financial resources for allocation to the various public welfare services. In this section mathematical programming techniques that have been used to help move toward solutions to some of these complex problems in local schools are reviewed.

Interest in programming models designed to increase the efficiency of local school district management and policy formulation is a relatively new concern. This is especially true in the areas of instructional policy formulation rather than in non-instructional problem areas such as school transportation and inventory control.

Benson (1967) attributes this lack of application to the fact that analytic tools of planning such as linear programming and simulation techniques are considerably better than the educational data to which they can be applied. More recently Lachene (1969) has noted that this gap should be attributed not only to educational data limitations but also to the nature of current educational planning theories.

In the specific case of mathematical programming, this gap is due in part to problems which confronts all programming models used to determine optimizing behavior in the public rather than the defense or industrial sector, namely, the absence of a production function designed to measure social systems outputs (Blumstein, Kamrass and Weiss, 1970). Educational production functions are necessary to form the objective function of these models.

Bowles (1970b) has noted

> In setting school policy and in long-range educational planning, knowledge of the educational production function is essential to efficient resource allocation. This is true whether the decision unit is pursuing the objective of growth or of equality, or a combination of these and other (perhaps noneconomic) goals. Without an estimate of the technology of education (the production function) the relationship between the opportunity cost and expected benefits of particular policies must be little more than guesswork (pg. 12).

> Moreover, the school output is multidimensional, and the relative valuation of different outputs--say, mathematical competence as opposed to citizenship--differs among school districts. For this reason, technical inefficiency may result neither from inadvertence nor from the absence of optimizing behavior but rather from the conscious pursuit of objectives not adequately measured in any single index of school output (pg. 17).

Thomas (1969) has constructed an "Administrator's Production Function" set in a mathematical programming format. This analytic approach to school management allows the administrator to increase his knowledge about the educational production function in order that he may wisely allocate the resources (including goods and time) which are at his disposal (p.8). Input-output studies, based on large cross-sectional statistical analysis, have provided some empirical guidelines for the construction of production functions (objective function) and for improvement of resource allocations (Burkhead, 1968; Levin, 1970). Other recent evidence of local level administrative interest in school production relationships include the findings reported by the AASA Commission on Administrative Technology (Knezevich, 1969) and by James (1968).

The local level applications reported here are grouped by topic areas. First, models relating directly to instructional policy planning for local schools are given. These are followed by applications that deal with important aspects of school administration but focus on problems that are only indirectly related to instructional planning.

Curriculum Planning

A number of recent mathematical studies have been concerned with the economic benefits of formal education to the individual and to the total society (Witmer, 1970; Levin, 1970). Few mathematical programming models have been utilized to determine the gains of specific curricular contents for the individual student or to a total school program.

Economists, who are concerned with this micro level of educational planning, usually develop programming models which assume that school systems are trying to maximize the net additional lifetime earnings of their graduates (Holtmann, 1968). Their concerns center on relating earnings to level of education completed, i.e., Grades 9, 10, 11, etc. Hence differentiation of graduates by specific curricular programs are seldom offered (Correa, 1964). Illustrations of how linear programming can be used to decide between general or vocational education have been proposed (Correa, 1965; Corazzini and Bartell, 1965), but little evidence of their subsequent use is available. Shina, Gupta, and Sisson (1969) have proposed two mathematical models which attempt to relate student achievement in schools to resource allocations. In these models Lagrangian multiplier techniques are used to determine an optimal resource allocation strategy.

Taft and Reisman (1967) have noted that recent major studies of curricular have concentrated on the content and method of presentation but have not made significant strides toward systematic exploitation of the potential contribution made by the learning process of reinforcement through proper sequencing of subject matter (p. 926). They provide a curriculum development model that uses a heuristic algorithm in order to optimize the sequencing of subject matter. Sensitivity analysis and simulation are used to check different schedule combinations generated for particular levels of mastery. They suggest that such a model can be used at all levels of the educational system from elementary through

graduate school, in highly academic as well as vocational training programs.

Decisions about patterns for grouping students, assigning staff members, and selecting programs are central to curriculum planning in school systems. Nystrand and Bertolart (1967) have observed that the most extensive work on coordinated planning has been in the area of master schedule building. The University of Pittsburgh project report includes a number of algorithmic, linear programming and heuristic models for master scheduling. In this report Harding (1964) has shown that linear programming techniques for master schedule building can be used to determine "shadow price" information. These data constitute a means for evaluating the marginal returns for potential changes in physical plant, personnel resources, student curriculum demands and other instructional policies.

Shapley, Fulkerson, Havelick and Weiler (1966) have used the transportation algorithm, a special form of linear programming, to solve the problem of utilizing idle classrooms in Los Angeles. Lawrie (1969) developed an integer linear programming model for the master schedule building. Turksen and Holzman (1970c) have recently shown how other optimization techniques, including branch and bound methods, can be used to solve scheduling problems where feasible scheduling spaces are represented by sets of boolean lattice points.

Salary Schedules and Price Indexes

Bruno (1969b) has used linear programming to develop an alternative approach to the fixed step salary schedule used by most school districts. The model is capable of incorporating many factors which

are considered important by teachers unions, school boards, etc., in salary evaluation. Hence, it might be used as a tool for collective bargaining negotiations with teacher unions. Nine factors are used to determine a flexible salary schedule which permits overlaps in the established salary hirearchy, i.e., it would permit highly qualified teachers to receive higher salaries than low qualified administrators (Bruno, 1969d).

Wasserman (1963) has shown that mathematical programming can be used to compile an education price index based on index number theory. Constrasts between (1) linear programming and indifference curve procedures, and (2) the relationship between the deterministic nature of linear programming and the random factors of other statistical models having specified probability density functions are briefly explained. This work provides useful insights into some of the problems involved in posing quantitative relations between educational inputs and education return.

Demographic Applications

Since the landmark Supreme Court decision in 1954 invalidating school segregation, the problem of racial desegregation of school systems has received a great deal of attention. In urban communities, where residential patterns produce *defacto* segregation of schools, many school administrations have adopted an official policy of eliminating such segregation. A number of recent studies have been conducted to determine differences in achievement that follow from implementing administrative policies regarding desegregation (St. John, 1970). Operations researchers

and model builders have also made a singificant contribution toward solving problems of desegregation in the schools. They have attempted to utilize transportation algorithms and other geographic optimization techniques as a means to solve the problem of residential segregation on the basis of ethnicity of students.

Clarke and Surkis (1968) have developed a system called "MINTRAN" which solves the school desegregation problem as a multi-product distribution problem. Given (1) the distribution of students in a community by race, (2) the location and capacity of each school, (3) the ethnic composition desired at each school, and (4) the configuration of mass transportation lines in the community; their system searches for a plan of assignment to schools which achieves (1) the desired ethnic composition at each school, (2) requires no student to travel more than a specific number of minutes per day, and (3) minimizes the total daily student travel time.

In a similar study Heckman and Taylor (1969) discuss some technical aspects of the linear programming, including the formulation of the problem, some alternative constraint possibilities, branch and bound techniques to avoid splitting neighborhoods and improvement in execution speed. A non-technical presentation of their ideas on designing school attendance zones appears in Heckman and Taylor (1970).

The operations research group at the University of Oklahoma in conjunction with the Oklahoma City Public School District have developed a model for school desegregation (Lutz, 1970). Based on constraints supplied by individual school administrators, best time periods for one-

section classes in a modified modular scheduling environment are determined. Although their systems can be modeled as a 0 - 1 quadratic programming problem, it is not feasible from a computational standpoint to solve this programming model exactly. A computer program is written to provide an efficient, though not necessarily optimal, solution.

O'Brien (1969) has provided a model to be used in the evaluation of educational policy relative to the location and size of school plants within an urban environment. Although specific school characteristics and demographic considerations are set in mathematical relationships, no attempt is made to maximize any specific alternatives relative to a defined utility function.

Ploughman (1968) has developed an assignment program for the Bloomfield Hills School District (Michigan) which provides the following information: (1) probable pupil population growth within the district, (2) desirable building sites selected from potential sites currently owned or available to the district, (3) preferred construction schedules for new schools, and (4) the assignment of geographic areas within the district to particular schools. Newton and Thomas (1969) have used optimization techniques to design school bus routes and schedules for a single district. Their work utilized a constraint network similar to those designed by Clarke and Surkis or Heckman and Taylor.

A final application illustrates the range of school problems to which mathematical programming can be applied. Lutz (1968) has used dynamic programming to construct a computer-assisted cafeteria menu plan. Based on the classic diet problem, a monthly school menu is produced which meets all of the legal requirements set forth in the U. S. Department of

Agriculture school lunch program. Provisions are also made to allow for ethnic considerations in selecting food items of equal nutritional value and acquisition cost.

Development of mathematical programming models for the local school district activities almost exclusively originates in disciplines other than education, i.e., industrial engineering, economics, business and operations research. Educators should continue to encourage this type of activity. However, the value of these models could be greatly enhanced if educational researchers and administrators would assist model builders to interpret the realistic nature of their model assumptions in light of the organizational characteristics of schools. For example, a given orthogonal variable, derived from a factor analytic study of an educational system, is often composed of a variety of components in a way that defies intuitive understanding of what the variable represents (Mood, 1969)

FUTURE DIRECTIONS

The scope of the review prohibits a detailed listing of future directions for each problem area. To compensate for this a number of research implications based on present findings have been included in the specific topic area reviews. Therefore, the final section includes some general comments relating to the future application of mathematical programming in educational planning.

Excellent guidelines for the future development of management science models for educational planning have appeared in the educational literature

(Hartley, 1969; U. S. Department of Health, Education and Welfare, 1970) and in other sources (Ackoff, 1969; Durstine, 1969; Feld, 1970). Fox and Sengupta (1968) and Weathersby (1970a) have outlined specific criteria for the construction of mathematical programming and simulation models in education. The classic works, dealing with the relations of judgments to optimal decisions, still provide the best references for construction of variables and parameters to be used in "decision-oriented" research in education (Ackoff, 1962). These works include current concerns such as (1) individual and collective value judgments, and (2) subjective programming (Shelly and Bryan, 1964).

The author has recently received (November, 1970) along with numerous research reports sixty replies from researchers (in various academic departments and private firms) who can be identified with programming models in educational planning. A few of their comments and current concerns for future directions of this activity are offered below.

1. Mathematical applications to management should stay clear of large general models and concentrate upon specific problem areas. The mistake that educators have made for years involves trying to solve huge problems that do not have handles.

2. If the general area of operations analysis is to preserve in educational management, then the appropriate approach seems to be the consistent application of these techniques to small problems. In that way, we will efficiently solve the large problems.

3. Distinctions between linear programming as it applies to developments in programmed instruction or research in optimization models for management should be made explicit to increase the efficiency of information retrieval languages in education.

4. Mathematical programming models can be used as one of the technical decision-making components of the broad-based Program-Planning-Budgeting and management information systems now emerging in many educational organizations throughout the country.

Sisson (1970) and the Government Studies Center (1969) have suggested how the simplex algorithm and integer mathematical programming might be used to generate appropriate utility indicators for Educational PBBS systems. A number of successful applications of mathematical programming with accounting data systems have appeared in the literature (Denski, 1967; Rappaport, 1969).

SUMMARY REMARKS

It should be emphasized that mathematical programming models do not make decisions, nor can they replace judgment on the part of the decision-maker. These models are designed to aid and support decision-makers by providing pertinent data on alternative programs and courses of actions. Planning models allow decision-makers to form expectations of future consequences, these expectations being based upon known empirical relationships and the decision-makers own judgments.

To the model builder and the decision-maker interested in mathematical programming in education, it might be mentioned that the risks of innovation were well understood by Machiavelli. He described them in "The Prince" as follows: "There is nothing more difficult to take in hand, more perilous to conduct, or more uncertain in its success than to take the lead in the introduction of a new order of things, because the innovator has for enemies all those who have done well under the old conditions and lukewarm defenders in those who may do well under the new."

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